Quantum Physics 1
Class 17

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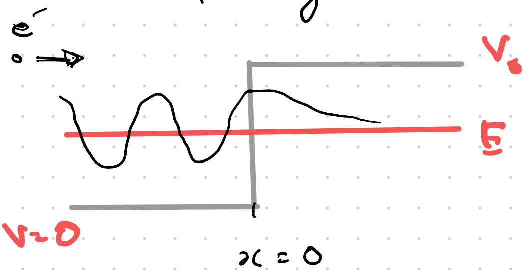
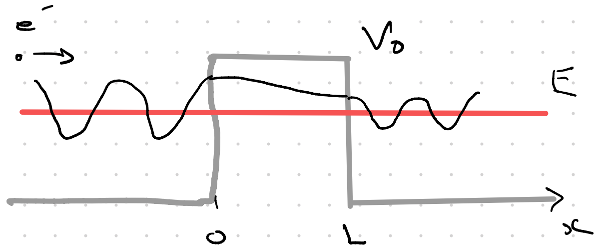
Principles of Quantum Physics

Review: Scattering & Bound states.

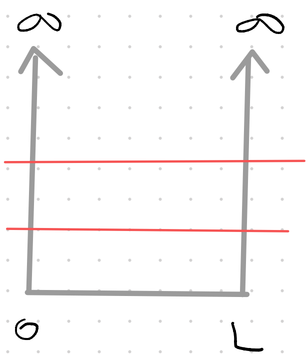
Scattering:



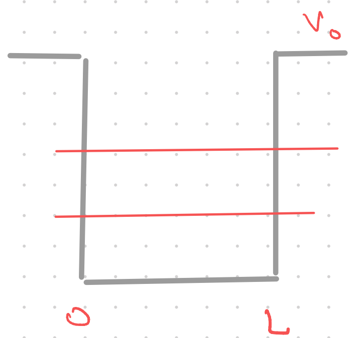


Bound states:

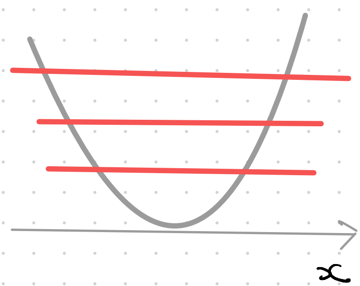
Infinite "□" well



Finite "□" well



Quantum Harmonic Oscillator



Solved the time independent S.E.,

$\psi_n(x)$, where $\hat{H} = \dfrac{\hat{p}^2}{2m} + V(x)$

$$\hat{H}\,\varphi_n(x) = E_n\,\varphi_n(x)$$

$$\psi(x) = \sum c_n\,\varphi_n(x) \quad ; \quad |c_n|^2 \sim \text{probability}$$

Quantum Postulates:

(1) Every measurable quantity "a" is associated with an operator $\hat{A}$. $\hat{A}$ is also called an "observable".

$$\hat{A}\,\varphi(x) = a\,\varphi(x)$$

eigenvalue

examples: $\hat{x}$, $\hat{p}$, $\hat{H}$, $\hat{L}$, $\hat{\Pi}$, ...

(2) Any measurement of A gives a corresponding eigenvalue a and the matching eigenstate, φ_n.

(3) $\Psi_{(x,t)}$ contains all the information of the system.

$|\Psi|^2 \sim$ probability density.

& $\langle A \rangle = \int \Psi^* \hat{A} \Psi \, dx$; expectation value of A.

(4) Time dependent $\Psi(x,t)$ obeys.

S.E. :

$$\frac{\hat{p}^2}{2m} \Psi_{(x,t)} + V_{(x)} \Psi_{(x,t)} = i\hbar \frac{\partial}{\partial t} \Psi(x,t)$$

(5) $\exists$ Identical particles

ex for e^-:

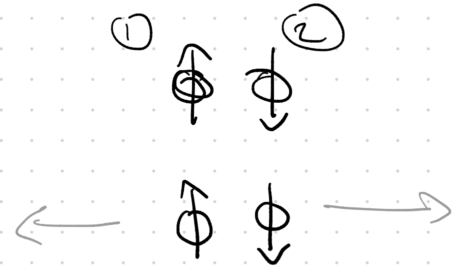
$$\Psi_A = \frac{1}{\sqrt{2}} \left\{ \varphi_n(1) \varphi_{n'}(2) - \varphi_n(2) \varphi_{n'}(1) \right\}$$

$$\Psi_S = \frac{1}{\sqrt{2}} \left\{ \varphi_n(1) \varphi_{n'}(2) + \varphi_n(2) \varphi_{n'}(1) \right\}$$

Gives rise to effects like:

(i) Entanglement (Prof. N'gom's research)

(ii) Teleportation.



make a meas.

Spin of 1 will be determined if 2 is measured.

Now, $\hat{H}\varphi_n = E_n \varphi_n$

1

& $\psi(x) = \sum c_n \varphi_n(x)$

also, consider the momentum operator:

2 $\frac{\hbar}{i}\frac{\partial}{\partial x}$

$\Rightarrow \frac{\hbar}{i}\frac{\partial}{\partial x}\varphi(x) = p\varphi(x)$; w/t solution of e^{ikx} independ. of $V(x)$.

$\frac{\hbar}{i}(ik)e^{ikx} = pe^{ikx}$

$$\boxed{p = \hbar k}$$

In-class 17.1, 17.2

Consider Parity operator $\hat{\Pi}$:

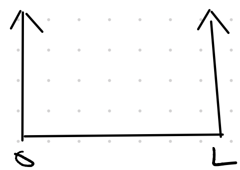
$$\hat{\Pi}\,\varphi(x) = \varphi(-x), \quad \hat{\Pi}\,\varphi(x) = \underset{?}{\underset{=}{\lambda}}\,\varphi(x)$$

$$\Rightarrow \quad \hat{\Pi}^2\varphi(x) = \hat{\Pi}\,\lambda\,\varphi(x)$$

$$= \lambda^2\varphi(x)$$

$$\therefore \quad \lambda^2 = 1, \quad \lambda = \pm 1$$

Aside:



$$\varphi_n = \sqrt{\frac{2}{L}}\,\sin\left(\frac{n\pi x}{L}\right)$$

$$\hat{\Pi}\sqrt{\frac{2}{L}}\,\sin\left(\frac{n\pi x}{L}\right) = 0$$

In-class 17-3

Hermitian Operators:

If an operator has:

(i) real eigenvalues

(ii) eigenfunctions are orthogonal & have distinct eigenvalues.

(iii) eigenfunctions form a complete set, ie. can express any function as a superposition of eigenfunctions

then the operator is called a Hermitian operator.

<u>NB</u>: Operators linked to observables are Hermitian.

Consider the following:

$$\int_{-\infty}^{\infty} \varphi^* (A \psi)\, dx = \int_{-\infty}^{\infty} (A\varphi)^* \psi \, dx$$

$$= \int_{-\infty}^{\infty} \varphi^* A^{\dagger} \psi \, dx$$

Consider $\varphi = \Psi$

$\Rightarrow \int \Psi^* (A\Psi)\, dx = \int (A\Psi)^* \Psi \, dx$

$\Rightarrow \langle A \rangle = \langle A \rangle^*$

Since $A\Psi = a\Psi$ where "a" is real

then $\langle A \rangle$ is real & $\langle A \rangle^*$ is also real.

** Necessary condition for an observable.

Now consider an operator D, $D = i \frac{d}{dx}$;

what is D^{+}?

$\Rightarrow \int \Psi^* D^{+} \varphi(x)\, dx = \int \left(i \frac{d}{dx} \Psi\right)^* \varphi \, dx$

[apply integration by parts, (see the textbook)]

$\Rightarrow \int \Psi^* \left(i \frac{d}{dx}\right) \Psi \, dx \Rightarrow \boxed{D^{+} = D}$

In-class 17.4

Matrix Mechanics / Representation

Recall: $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

inner product : $\vec{A} \cdot \vec{B} = (A_x \; A_y \; A_z) \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}$

$= A_x B_x + A_y B_y + A_z B_z$

So if $\Psi_A(x) = \sum a_n \varphi_n(x)$; $\Psi_B(x) = \sum b_n \varphi_n(x)$

inner product : $(a_1^* \; a_2^* \; \ldots) \begin{pmatrix} b_1 \\ b_2 \\ \vdots \end{pmatrix} = \sum_n a_n^* b_n$

now,

$(A) \Rightarrow \begin{pmatrix} A_{11} & A_{12} & A_{13} & \cdots \\ A_{21} & A_{22} & & \ddots \\ \vdots & & & \end{pmatrix}$; $A_{nn'} = \int \varphi_n^* A \varphi_{n'} \, dx$

$\langle A \rangle = \int \Psi^* \hat{A} \Psi \, dx = (c_1^* \; c_2^* \ldots) \begin{pmatrix} A_{11} & A_{12} & \cdots \\ A_{21} & \ddots & \\ \vdots & & \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ \vdots \end{pmatrix}$