Quantum Physi cs 1
Class 16

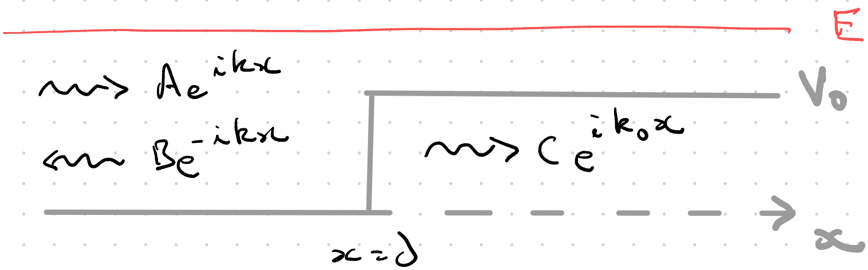
Class 16

Quantum Tunneling

Last time:

Scattering from a Step Potential

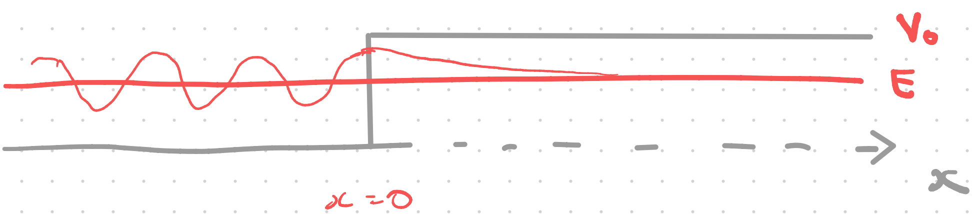
$E > V_0$



Consider the probability flux j:

$$
\left.
\begin{aligned}
j_A &= \frac{\hbar k}{m} AA^* \\
j_B &= \frac{-\hbar k}{m} BB^* \\
j_C &= \frac{\hbar k_0}{m} CC^*
\end{aligned}
\right\}
\qquad
\begin{aligned}
&x<0 \quad k = \sqrt{\frac{2mE}{\hbar^2}} \\
&x>0 \quad k_0 = \sqrt{\frac{2m(E-V_0)}{\hbar^2}} \\
&\text{Can calculate } R \text{ \& } T.
\end{aligned}
$$

For the case $E < V_0$



for $x > 0$: $k_0 = \sqrt{\dfrac{2m(E-V_0)}{\hbar^2}}$

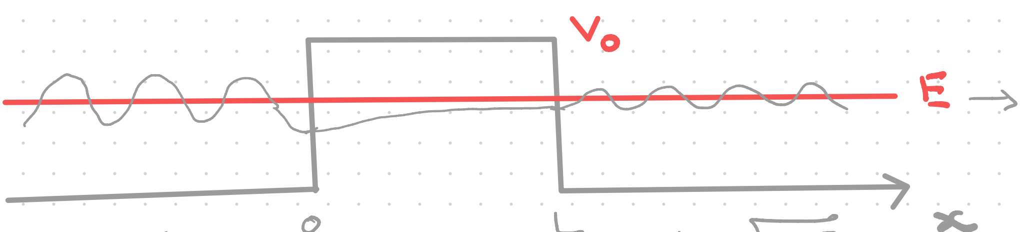
$= i\sqrt{\dfrac{2m(V_0-E)}{\hbar^2}}$

$= iK_1$

w/ solutions :

on RHS : $Ce^{ik_0x} = Ce^{-K_1x}$;

Now in the case of finite barriers, if the width of the barrier < decay of Ψ then the Ψ can "escape" or "tunnel" to the RHS from the LHS.



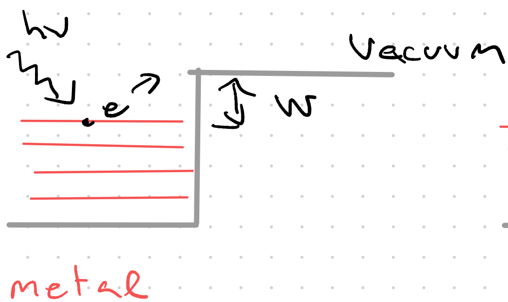
$k = \sqrt{\dfrac{2mE}{\hbar^2}}$ $\qquad$ $k = \sqrt{\dfrac{2mE}{\hbar^2}}$

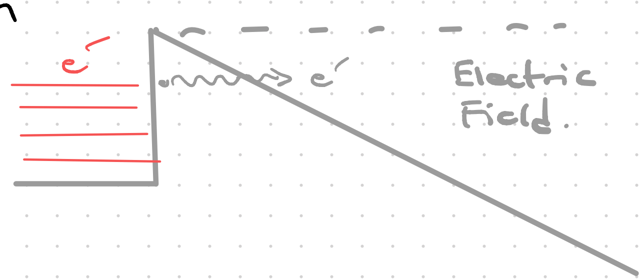
NB: The amount of tunneling depends on the height and the width of the barrier; V_0, L.

Applications:

1. Scanning Tunneling Microscopy (STM)
2. Field emission scanning electron microscopy, cold emission (SEM).
3. α-decay.

Recall Photoelectric effects:

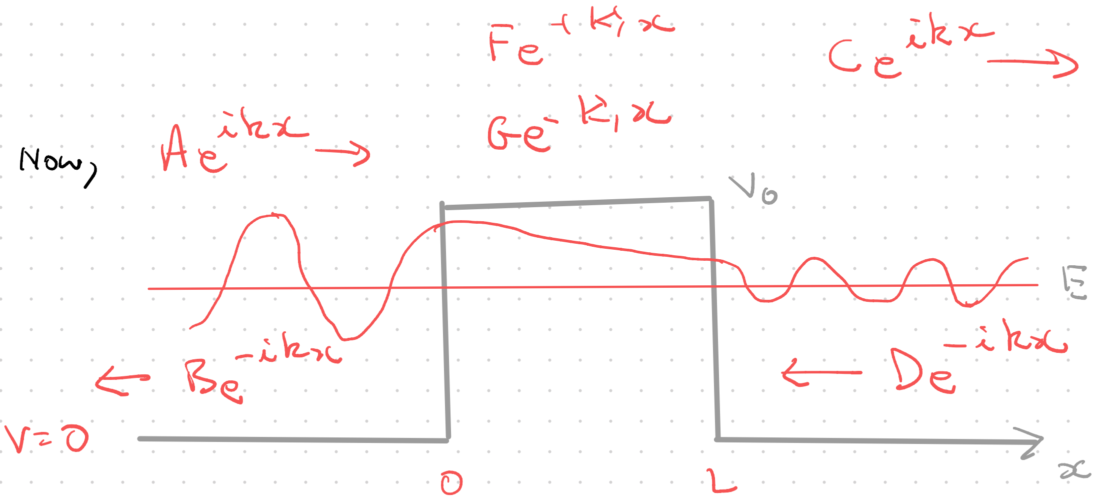




In-class #16-1

Now let's consider the finite barrier as a scattering problem. With an incident wave coming from the left hand side (LHS).

Now,



$$\left.\begin{aligned} x<0 &;\quad Ae^{+ikx} + Be^{-ikx} \\ 0<x<L &;\quad Fe^{k_1 x} + Ge^{-k_1 x} \\ x>L &;\quad Ce^{ikx} + \cancel{D}e^{-ikx} \end{aligned}\right\}$$

0

$$T = \frac{|C|^2}{|A|^2} = \left(1 + \frac{(k^2 + k_1^2)^2}{4k^2 k_1^2} \sinh^2 k_1 L\right)^{-1}$$

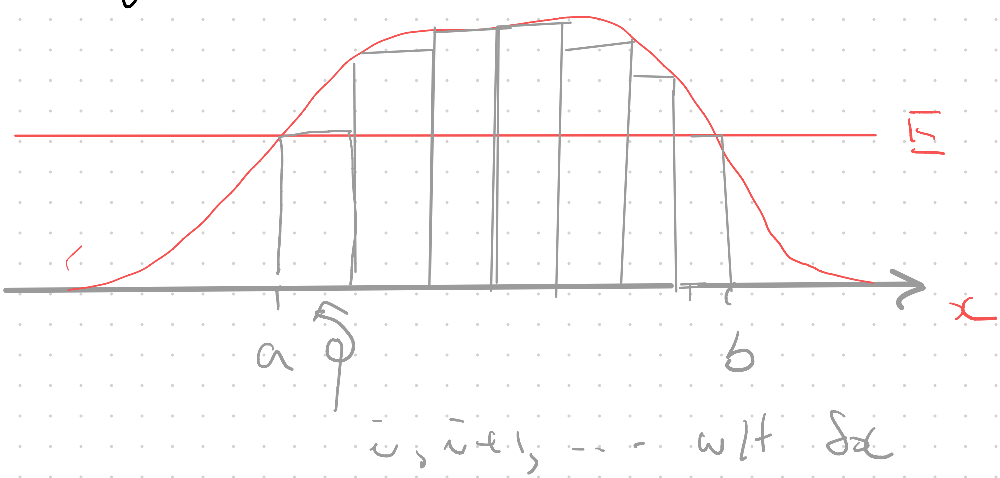
where $\sinh(k_1 L) = \dfrac{e^{k_1 L} - e^{-k_1 L}}{2}$

for $\kappa_1 L \gg 1$; $\sinh(\kappa_1 L) \sim \frac{1}{2} e^{\kappa_1 L}$

$$T = \left(\frac{4 k \kappa_1}{k^2 + \kappa_1^2} \right)^2 e^{-2\kappa_1 L}$$

In-class 16.2, 16.3, 16.4

Consider potential barrier with an arbitrary shape:



$$T = 16\,\frac{E}{V_0}\left(1-\frac{E}{V_0}\right) e^{-2K_1 L}$$

$\ln T = \text{const} - 2K_1 L$; w/t $2K_1 L \gg \text{const}$.

Now, $\ln T = \ln \prod_i T_i = \sum_i \ln T_i$

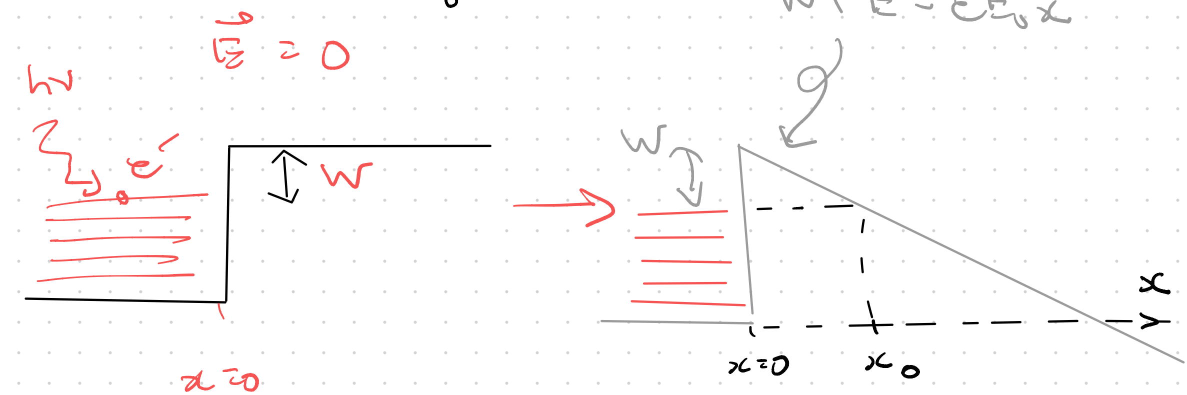
$$= -2\sum_i K_{1i}\,\delta x$$

recall: $\ln(A\cdot B) = \ln A + \ln B$.

$$\therefore\ T \approx C\, e^{-2\sum_i K_{1i}\,\delta x}$$

$$\boxed{T \approx C\, e^{-2\int_a^b dx \sqrt{\frac{2m(V_0-E)}{\hbar^2}}}}$$

NB: There are many instances where we need to consider a barrier that's not rectangular:



{Recall: Electric field, E_0 w/t $V = qE_0 x$ }

Now, $E = W + E - eE_0 x_0$

$$0 = W - eE_0 x_0$$

$$x_0 = W / eE_0$$

So,

$$T \approx C e^{-2\int_0^{x_0} dx \sqrt{\frac{2m}{\hbar^2}(W - eE_0 x)}}$$

$$\underbrace{\approx C e^{-\left(\frac{4}{3\hbar}\right)\sqrt{2m}/E_0 \, W^{3/2}}}_{\text{Fowler-Nordheim Relationship.}}$$