Quantum Physics 1
Class 13

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Simple Harmonic Oscillator

Review:

$\Psi(x), \varphi_n, \ldots$

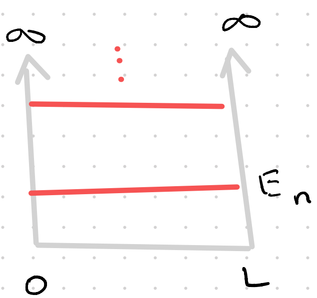
$\hat{x}, \hat{p}, \hat{H}, \ldots$

(i) Functional Vector Space.

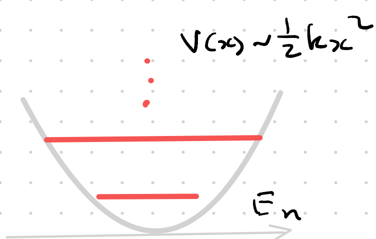
(ii) Energy eigenstates of the time independent S.E.

$$\underbrace{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\varphi_n + V(x)\,\varphi_n(x)}_{\hat{H} \,\equiv\, \text{Hamiltonian operator}} = E_n \varphi_n(x)$$

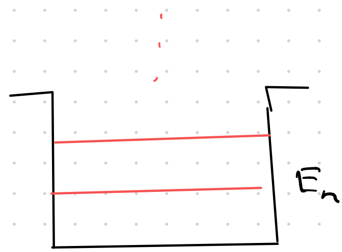
Examples of different eigenstates that manifest from different $V(x)$:



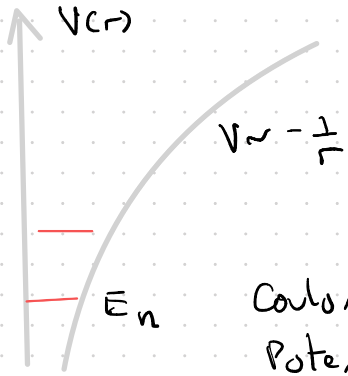
∞- square well



Harmonic Potential well



Finite square well



Coulomb Potential.

- Can write $\Psi(x) = \sum c_n \varphi_n(x)$, expansion of eigenstates of the $\hat{H}$. Solutions of time dependent S.E.:

$$\Psi(x,t) = \sum_n c_n \varphi_n(x) e^{-iE_n t/\hbar}$$

- Can solve fourier coefficients by considering

$$\int \varphi_m^* \Psi(x,t=0)\, dx = \int \sum c_n \varphi_m^*(x) \varphi_n(x)\, dx$$

$$= \sum c_n \delta_{mn}$$

$$= c_n$$

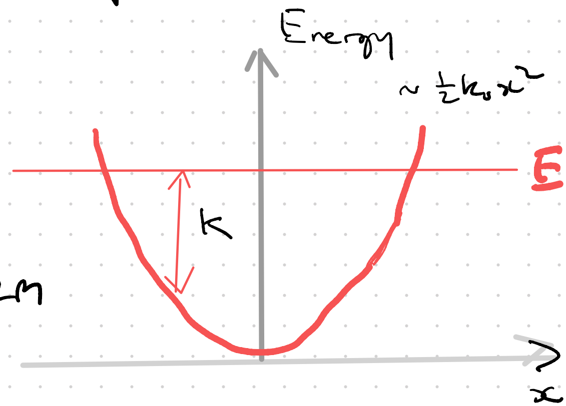
Recall: Classical Harmonic Oscillator

$V = \frac{1}{2} k_o x^2$, $k_o \equiv$ spring constant

$$F = -\frac{\partial V}{\partial x} = -k_o x = m \frac{d^2 x}{dt^2}$$

with solutions : $e^{i\omega t}$; $\omega \equiv \sqrt{k_o/m}$

& $H = \frac{p^2}{2m} + V(x) \Rightarrow$



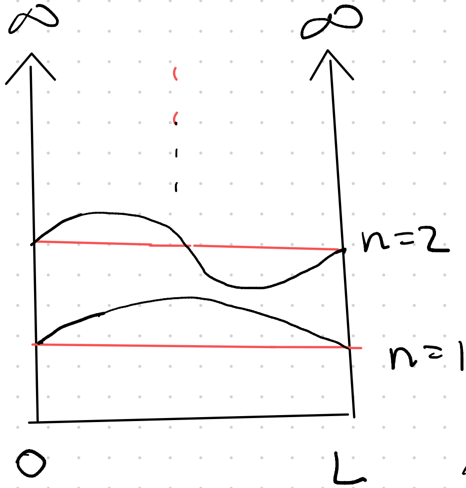
$K = p^2/2m$

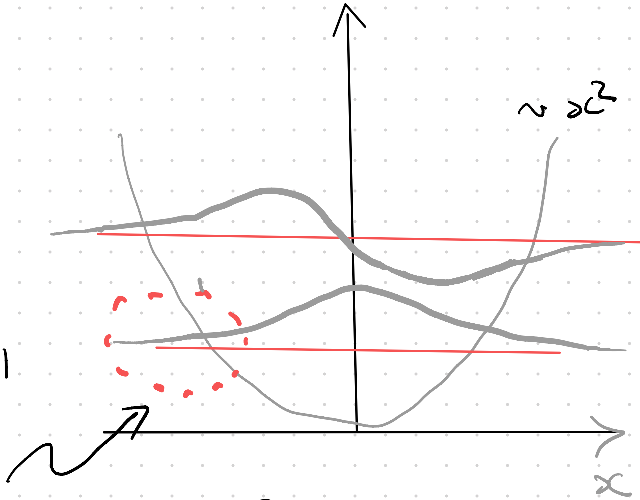
$\therefore$ $E = \frac{p^2}{2m} + V(x)$

Quantum Case:

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x}) \Rightarrow -\frac{\hbar^2}{2m} \frac{\partial^2 \varphi(x)}{\partial x^2} + V(x)\varphi(x) = E\varphi(x)$$

$\boxed{\text{In-class } 13.1 , 13.2}$





∞ - square well

wave penetration

Quantum Harmonic Oscillator (QHO)

Soln: e^{ikx}

for $\left[\frac{p^2}{2m} + V(x)\right]\varphi = E\varphi$

= 0 inside, ∞ outside

For QHO $V(x) \sim \frac{1}{2}kx^2$

Eigenfunctions for the QHO

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\varphi(x) + \frac{1}{2}k_0 x^2 \varphi(x) = E\varphi(x)$$

with $\varphi_n(x) = A_n H_n\left(\sqrt{\frac{m\omega}{\hbar}}\, x\right) e^{-m\omega x^2/2\hbar}$

$$\boxed{\begin{array}{l}\text{Hermite Polynomials} \\ H_n(y) = (-1)^n e^{y^2} \dfrac{d^n}{dy^n} e^{-y^2}\end{array}}$$

$E_n = (n + \frac{1}{2})\hbar\omega \; ; \quad n = 0, 1, 2, \ldots$

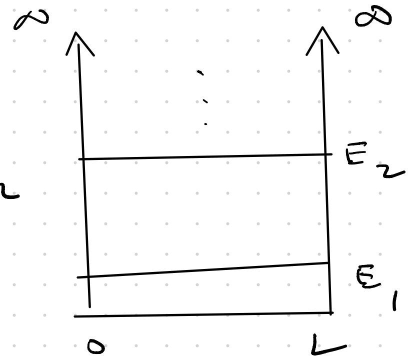
$$\boxed{\begin{array}{l}\underline{\text{NB}}: \quad n = 0 \quad \text{Ground state} \\ \text{w/ } E_0 = \frac{1}{2}\hbar\omega\end{array}}$$

$\Delta E_{n,n+1} = \hbar\omega \; ; \quad E_{n+1} - E_n = [(n+1) + \frac{1}{2}]\hbar\omega - [n + 1]\hbar\omega$

$= \hbar\omega$

$\boxed{\text{In-class 13.3, 13.4}}$

Compare the Infinite Square Well:



$$\Delta E_{n+1,n} = E_{n+1} - E_n$$

$$= \frac{\hbar^2 (n+1)^2 \pi^2}{2mL^2} - \frac{\hbar^2 n^2 \pi^2}{2mL^2}$$

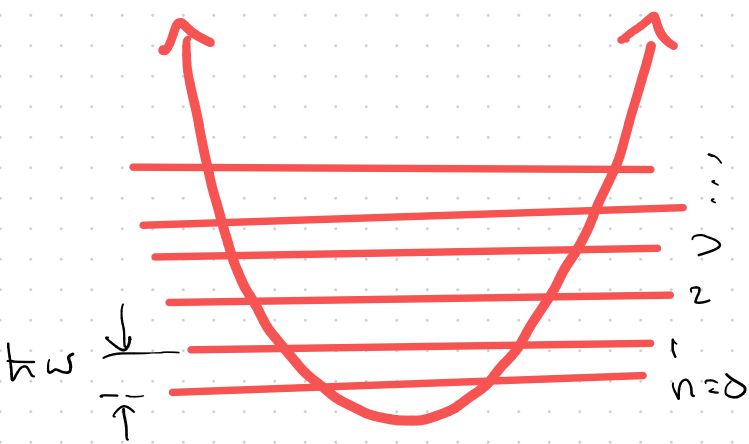
$$= \frac{2n\pi^2\hbar^2}{2mL^2} + \frac{\pi^2\hbar^2}{2mL^2}$$

$\propto n$ (dependent on n!)

$\therefore$ if $\Delta E_{n+1,n} \uparrow$ then $n \uparrow$

But for QHO: $\Delta E = \hbar\omega$;

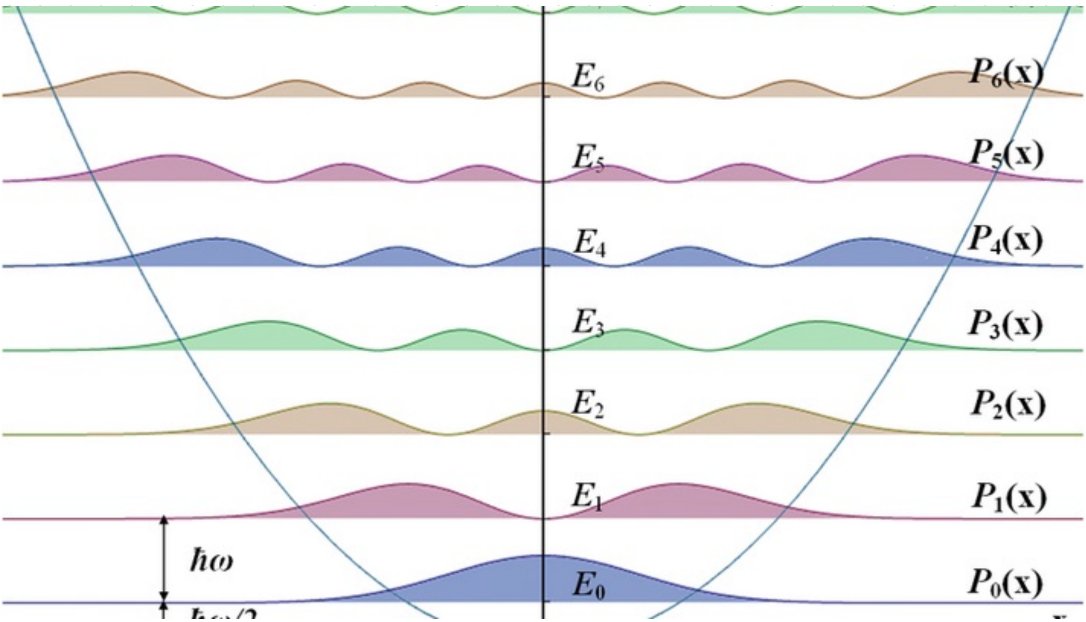
equally spaced and independent of n.



Applications :

1. Phonons
2. Molecular vibrations.

:. Any system where can make parabolic approximation at small E, V.



The Science Of Agents Of S.H.I.E.L.D.: What Is Quantum Harmonic Oscillation?

https://www.forbes.com/sites/chadorzel/2015/10/13/the-science-of-agents-of-s-h-i-e-l-d-what-is-quantum-harmonic-oscillation/#7c2b81c3e7e3