Physics 1250

Inductance

EMF and Current in Circuits

Magnetic Field Energy

Inductance

- Last class:
  - Faraday's Law – a time-changing magnetic flux induces an emf in a region of space, and can also induce current in a conductor located where the emf is occurring.

$$\oint \vec{E} \cdot d\vec{s} = -\frac{\partial \Phi_B}{\partial t}$$

- <u>Inductance</u> is the influence or effect that changing current or field exerts on a device by its own doing (self-inductance) or on other devices (mutual inductance).
  - An inductor is a device that functions in a circuit to mitigate rapid changes in current and to store magnetic field energy.

Inductance (2)

- Magnetic flux from a current i will generally have the form $\Phi_B = Gi$, where G is a factor that incorporates the geometry of a device or devices magnetically linked in some way (sometimes referred to as "flux linkage").
- G motivates the idea that there is a quantity known as inductance, which associates physical and geometric aspects of a conducting device with its magnetic properties.
- Self-inductance L of a device is defined by the relation

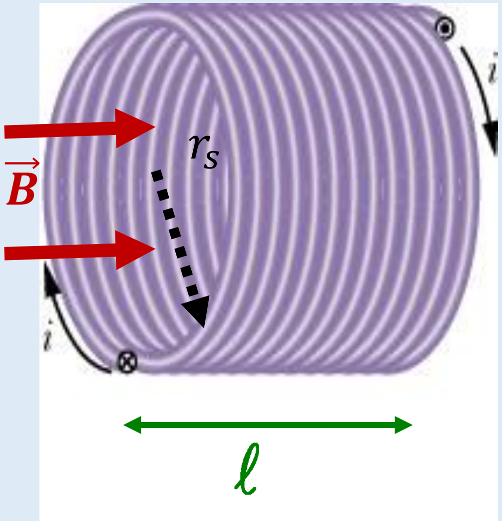
$$\Phi_B = Li \; ,$$

Φ_B = total magnetic flux contained within the current-carrying region of the device.

- Mutual inductance M relates the flux through one device to the current in another.

$$\Phi_1 = Mi_2$$

Example: Self-Inductance of a Solenoid



- Calculate the self-inductance (often just called the inductance) L of an ideal solenoid of N turns, length l, and radius r_s :

$$L_{oneturn} = \left|\frac{\Phi_{B,oneturn}}{i}\right| = \frac{\Phi_{B,turn}}{i} = \frac{BA_{turn}}{i}$$

$$L_{Nturns} = \frac{N(\mu_o ni)A_{loop}}{i}$$

$$\Rightarrow \; L_{culindrical\ solenoid} = \mu_0 N\left(\frac{N}{l}\right)A_{loop} = \frac{\mu_0 N^2 \pi r_s^2}{l} = \pi\mu_0 n^2 r_s^2 l \;\;,$$

$\Phi_{B,loop}$ = flux through a single turn, and

$A_{loop} = \pi r_s^2$ = area of a turn.

- L is <u>independent</u> of current or magnetic field of the solenoid.
- SI unit of inductance is the Henry (H)*:

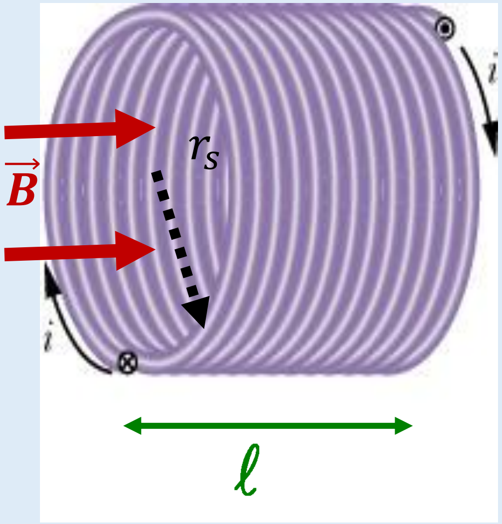
1 H = 1 Wb/A = 1 T m^2/A .

*ask for a Albany-Troy story

Example: Self-Inductance of a Solenoid (2)

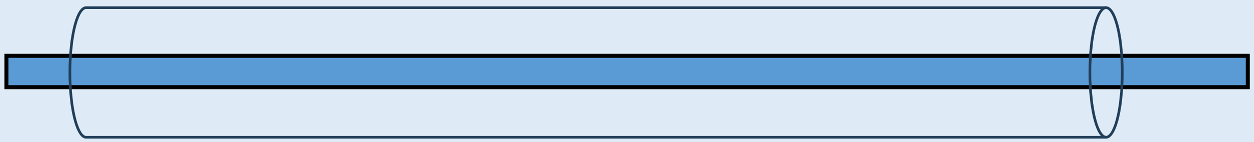
- To get a sense of a typical inductance value, an inductor in an electrical circuit might have 1000 turns, a radius of 0.50 cm, and a length of 1.0 cm:

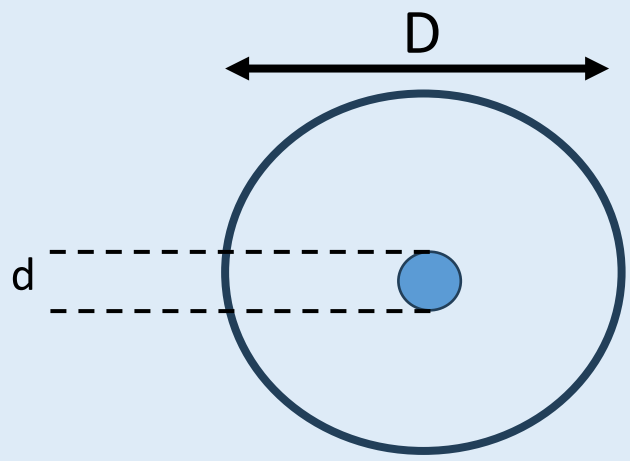
$$L = \frac{\pi\left(4\pi\times10^{-7}\frac{\mathrm{T\ m}}{\mathrm{A}}\right)(1000)^2}{(0.01\ \mathrm{m})}\ (0.0050\ \mathrm{m})^2 = 9.9\times10^{-3}\ \mathrm{H}.$$



Activity: Self-Inductance of a coaxial cable

- Determine the inductance of a one meter segment of a long straight coaxial cable. (See activity instructions)





- The inner wire diameter d is 0.80 mm. The diameter D of the inside surface of the outer conductor is 5.0 mm. The insulator is non-magnetic.

Inductance and EMF

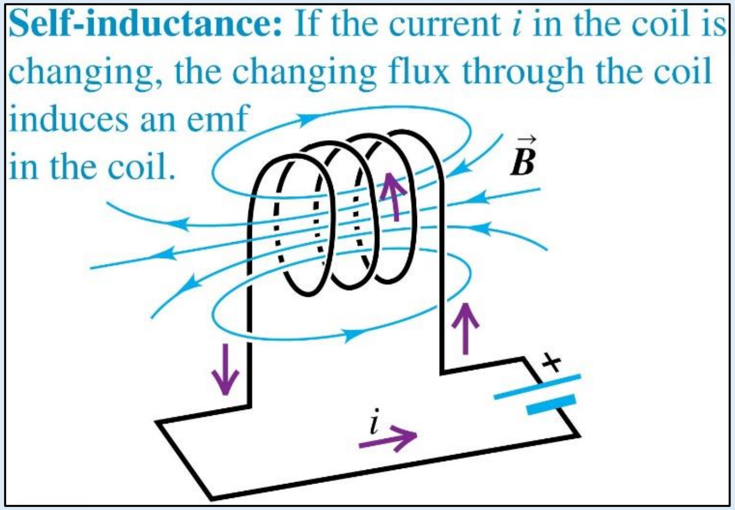
- Differentiating the inductance – current relation gives

$$\frac{d\Phi_B}{dt} = \frac{d}{dt}\,(Li) = L\,\frac{di}{dt}\,,$$

Combined with Faraday's law, yields:

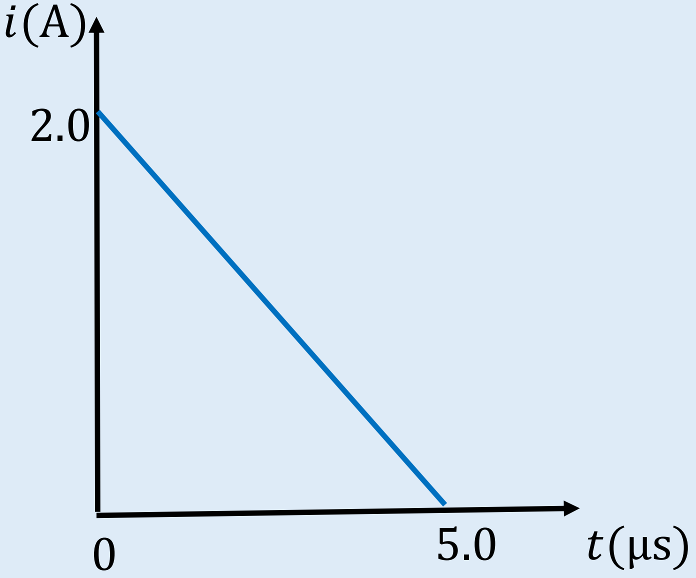
$$\mathcal{E}_L = -L\,\frac{di}{dt}\ .$$

- When current changes in an inductor, an emf is induced that opposes the imposed change (Lenz's law).
- Physical consequence: when current is turned on or off in a circuit (e.g., closing a switch), a "<u>back emf</u>" $\mathcal{E}_L$ is generated in the inductor that opposes that change. If the change in current is sudden, the back emf can be quite large in magnitude!



Lecture Question

- An ideal solenoid has length $l = 1.5$ cm, cross-sectional area $A_{loop} = 0.78\ \mathrm{cm}^2$, and number of turns $N = 2500$. At time $t = 0.0$ s a steady current $i_0 = 2.0$ A flows through the solenoid. Suppose the current were to be cutoff as shown in the graph at right. (This is consistent with pulling a power supply plug from the wall.) The magnitude of the emf induced in the inductor is closest to:



A. 1.2×10^5 V.

B. 1.3×10^3 V.

C. 21 V.

D. $\boxed{1.6 \times 10^4 \text{ V.}}$

E. 55 V.

$$L = \pi\mu_0 n^2 r_s^2 l = 0.041\ H$$

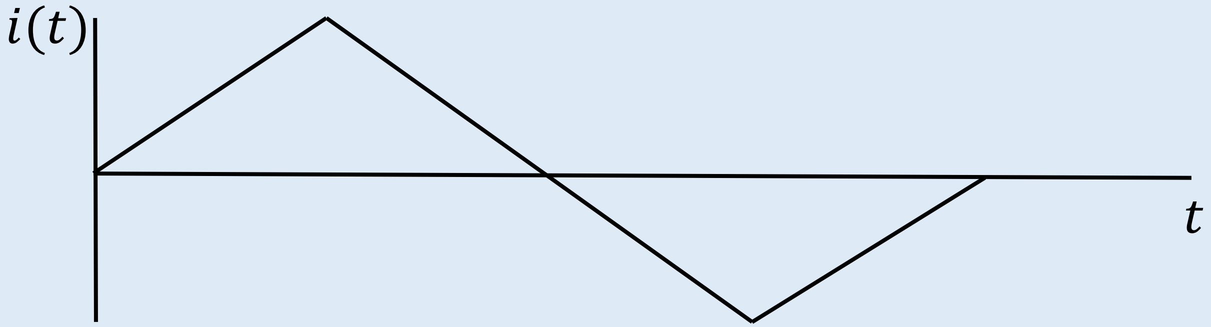
$$EMF = \left|-L\frac{\partial i}{dt}\right| = 0.041 * \frac{2.0}{5 \times 10^{-6}}$$

Inductors as Surge Protectors

- Because they oppose sudden changes in current, inductors can be used as surge protectors in electrical circuits and systems.
- Large inductors are used as surge protectors against lightning strikes for outdoor power transmission systems.

Calculus with an Inductor

- Because emf in an inductor is proportional to the time derivative of the current, can use this to perform basic calculus with inductors in circuits.
- Example: if the current in a circuit is a triangle wave, how does emf across the inductor behave?

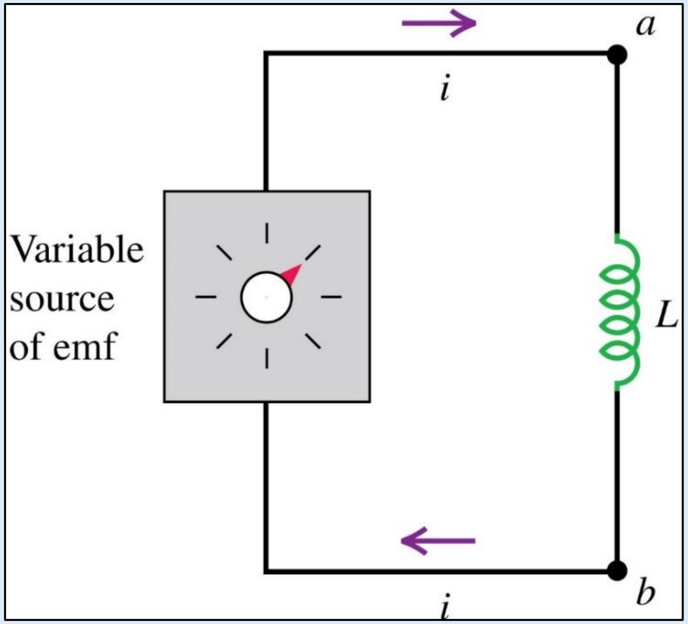


Change in Potential Across an Inductor

- Last class: Faraday's law relates change in magnetic flux to the line-integral of a non-conservative electric field $\vec{\boldsymbol{E}}_{nc}$:

$$\oint \vec{\boldsymbol{E}}_{nc} \cdot d\vec{\boldsymbol{l}} = -\frac{d\Phi_B}{dt}\,.$$

- Because the magnetic flux change in a circuit is confined within the inductor region, $\vec{\boldsymbol{E}}_{nc}$ is also confined between the terminals of the inductor (typically labeled points a and b in your text).



- Because $\vec{\boldsymbol{E}}_{nc} \neq 0$ is only between a and b, integrating the line integral in Faraday's law clockwise around the loop gives

$$\int_a^b \vec{\boldsymbol{E}}_{nc} \cdot d\vec{\boldsymbol{l}} = -\frac{d\Phi_B}{dt} = -L\frac{di}{dt}\ ,$$

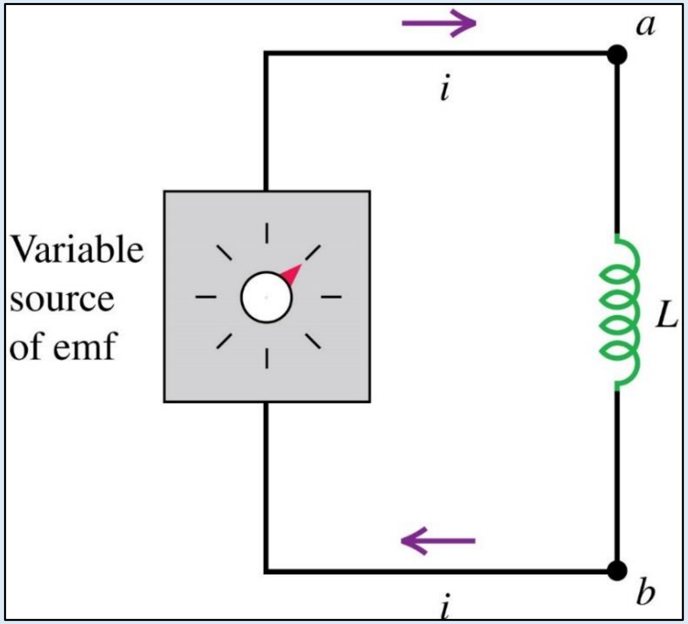
(used the inductance – flux definition presented earlier).

Change in Potential Across an Inductor (2)

- Ideal inductors are made of conductors with effectively zero resistance. To have finite current density in this type of inductor, the total electric field that acts on charges inside the conductor must be essentially zero:

$$\vec{\boldsymbol{E}}_{tot} = \vec{\boldsymbol{E}}_c + \vec{\boldsymbol{E}}_{nc} = 0 \quad \Rightarrow \vec{\boldsymbol{E}}_c = -\vec{\boldsymbol{E}}_{nc} \ .$$

- ∴ Conservative electric field $\vec{\boldsymbol{E}}_c$ in the inductor is opposite in direction and has same magnitude as the induced non-conservative field, $\vec{\boldsymbol{E}}_{nc}$.



- Electric potential is defined in terms of conservative electric fields; follows that potential difference between inductor leads is

$$V(b) - V(a) = -\int_a^b \vec{\boldsymbol{E}}_c \cdot d\vec{\boldsymbol{l}} = \int_a^b \vec{\boldsymbol{E}}_{nc} \cdot d\vec{\boldsymbol{l}} = -L\frac{di}{dt}$$

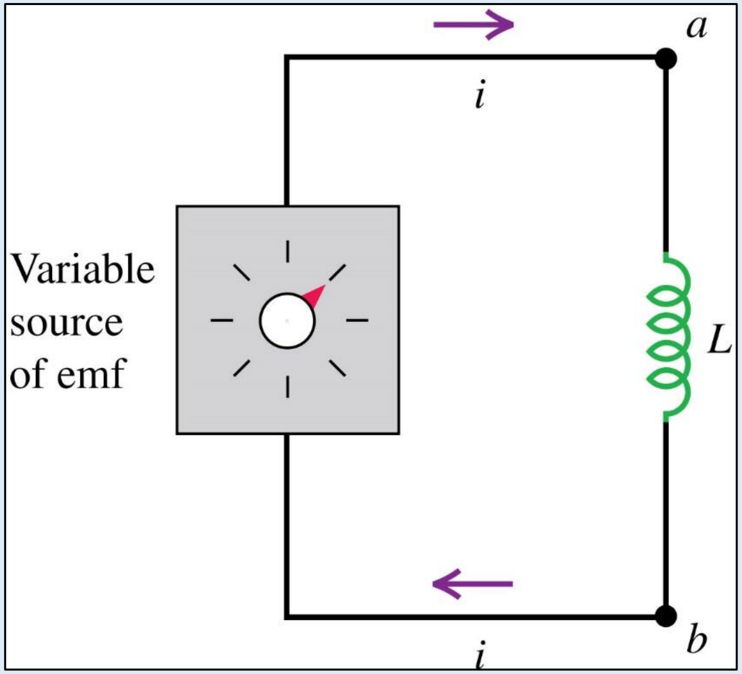
$$\Rightarrow V(a) - V(b) = L\frac{di}{dt} \ .$$

Inductors in Circuits

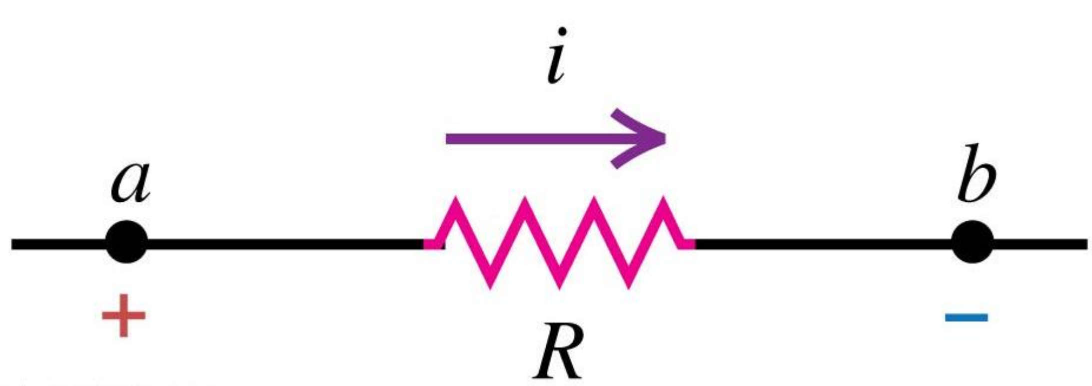
- Circuit shown: box enables us to control the current i in the circuit.
- Potential difference between the terminals of the inductor L is:

$$V_{ab} = V(a) - V(b) = L\frac{di}{dt}\ .$$

- Consider inductor response for different situations.



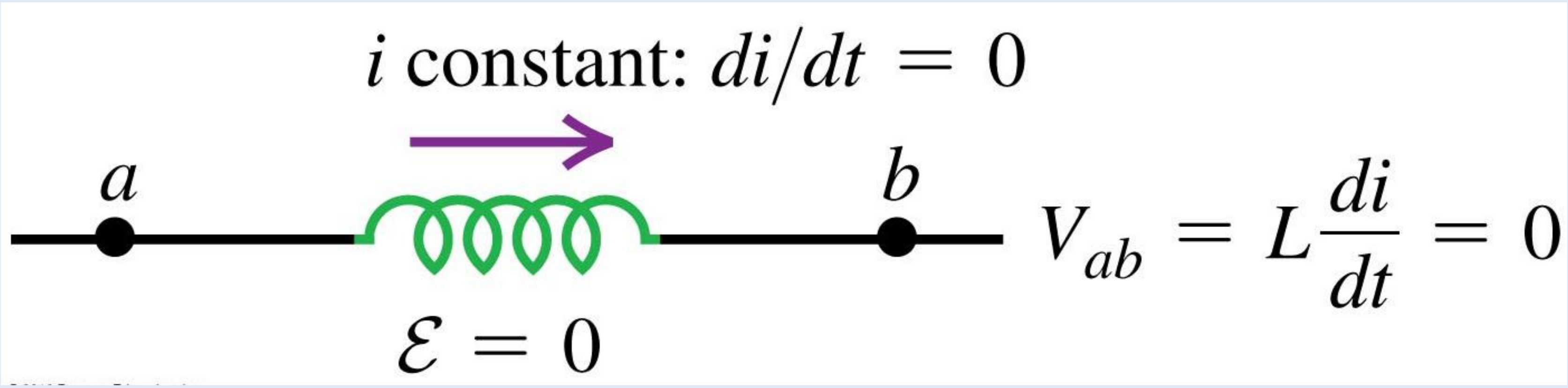
Inductors in Circuits (2)



$$V_{ab} = iR > 0$$

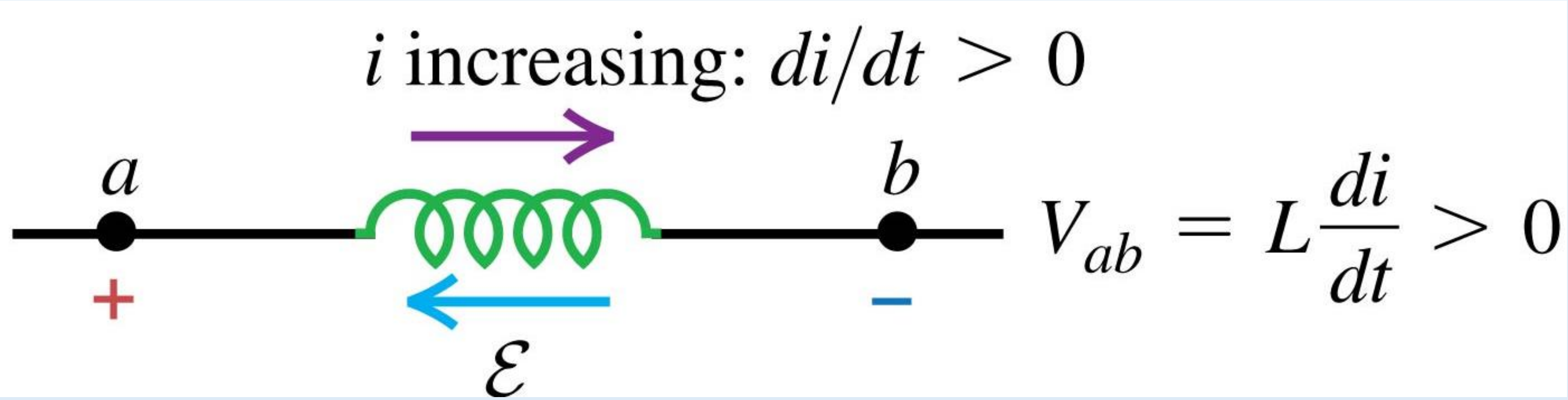
- For resistors in circuits, flow shown above is always a <u>drop</u> in potential across a resistor.
- Result for an inductor in a circuit will depend on how the current is <u>changing</u> with time.

Inductors in Circuits (3)



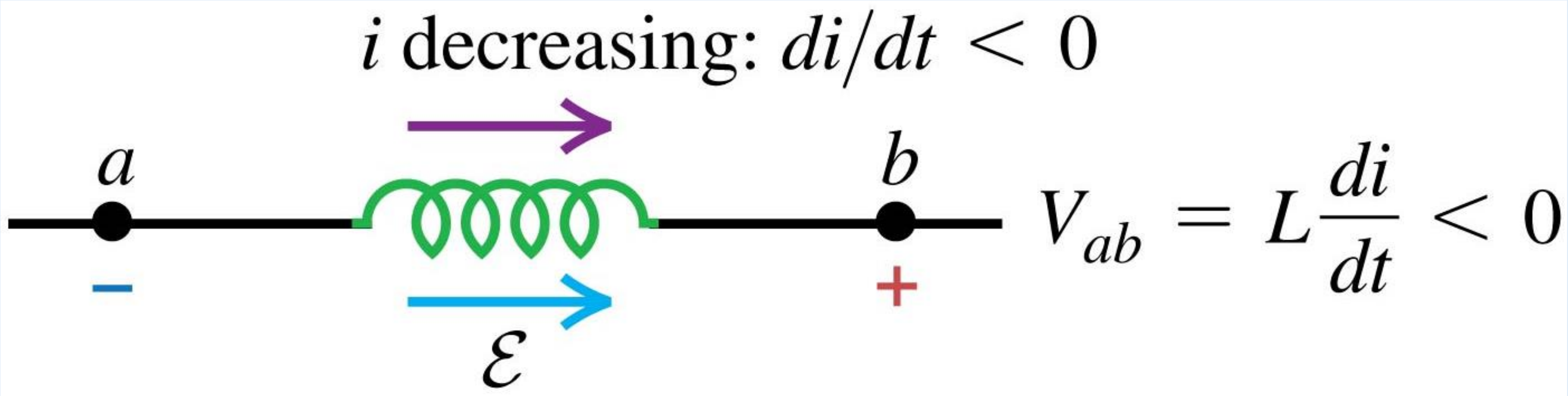
- Steady (i.e., constant) current: no emf and no potential difference across the inductor.

Inductors in Circuits (4)



- Current increasing in the direction of the current: induced back-emf in circuit opposes increase of the current in sense shown.
  - ➢ For this case, emf due to $\vec{\boldsymbol{E}}_{nc}$ in the inductor is directed from terminal b to terminal a.
  - ➢ Conservative electric field $\vec{\boldsymbol{E}}_c$ in the inductor is directed oppositely, from terminal a to terminal b.
  - ➢ Conservative electric fields point from high electric potential to low electric potential: follows that $V(a) > V(b) \Rightarrow V_{ab} > 0$.

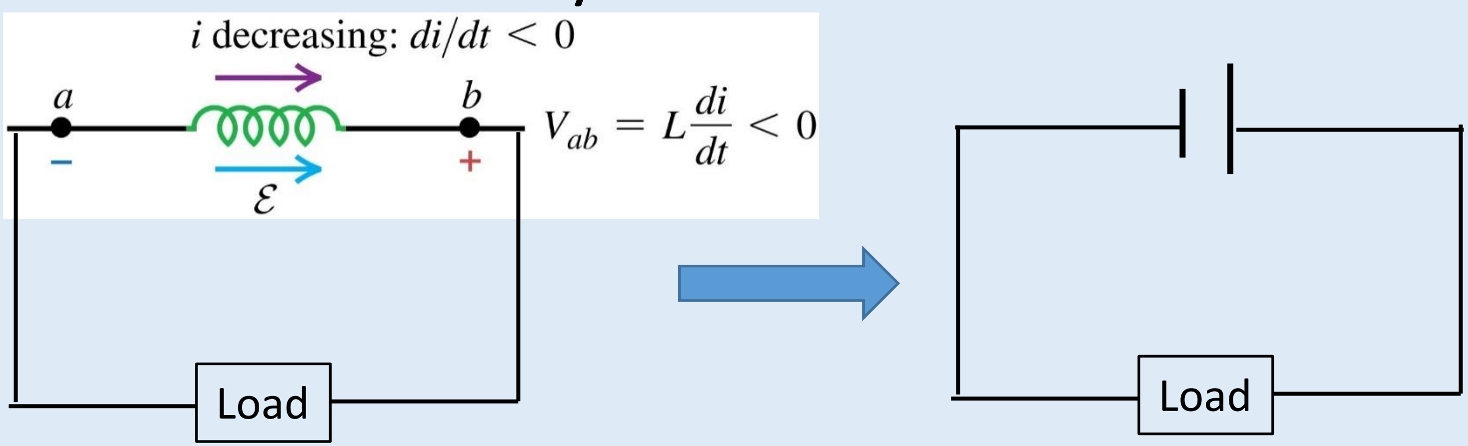
Inductors in Circuits (5)



- Current decreasing with time in the direction of the current.
  - Emf of the inductor tries to replenish (i.e., add to) diminishing current in the sense shown, going from a to b. $\therefore$ $\vec{\boldsymbol{E}}_{nc}$ for this case goes from a to b.
  - Conservative electric field $\vec{\boldsymbol{E}}_c$ is oppositely directed, from b to a.
  - Follows that $V(b) > V(a)$ for this case. That is, $V_{ab} < 0$.

Inductors in Circuits – Remembering the emf sign

- It can help to think of the potential induced by the changing current in the inductor as the potential necessary to maintain the current in the rest of the circuit.
- If current in the inductor is decreasing, the inductor will try to keep the current going in the rest of the circuit like a battery.



Magnetic Energy Stored in Inductors

- When inductor voltage $V_{ab} \neq 0$ and current $i \neq 0$, power can be supplied to an inductor. Amount of power will be

$$P_L = i\,V_{ab} = iL\frac{di}{dt}\,.$$

- Energy stored in the inductor as magnetic energy found by integrating supplied power over time:

$$U_B = \int P_L\,dt = \int iL\frac{di}{dt}dt = L\int_0^i i'di' = \frac{1}{2}\,Li^2 \quad .$$

  - Unlike a resistor, where energy is dissipated (i.e., lost from system), power P_L to the inductor is stored as energy in the magnetic field of the inductor.
  - If current in the circuit $i \to 0$, stored magnetic energy is returned to the circuit.

Energy Density of the Magnetic Field

- Similar to electrostatics, define the energy density of a magnetic field as

$$u_B \equiv \frac{U_B}{v} \text{ , where } v = \text{volume of a system.}$$

- For a solenoid of length l and cylindrical cross-section area A_{loop}, $v = A_{loop} l = \pi r_s^2 l$, $B = \mu_0 n i$, and $L = \pi \mu_0 n^2 r_s^2 l$. Follows that

$$u_B = \frac{\frac{1}{2} L i^2}{A_{loop} l} = \frac{1}{2} \frac{(\mu_0 \pi r_s^2 n^2 l) i^2}{\pi r_s^2 l} = \frac{1}{2} \mu_0 (ni)^2 = \frac{1}{2} \mu_0 \left(\frac{B}{\mu_0}\right)^2 ,$$

$$\Rightarrow u_B = \frac{B^2}{2 \mu_0} .$$

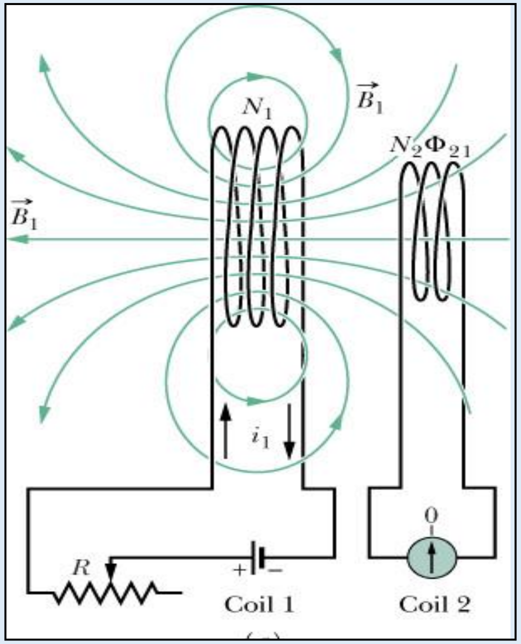
- Although derived for a solenoid, expression found for u_B is independent of any device. Above relation is true for all situations.

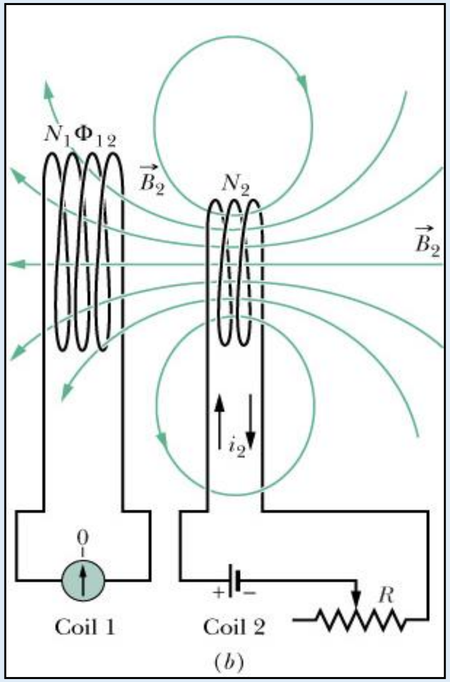
Energy Density of the Magnetic Field (2)

- Energy density u_B can be considered an intrinsic property of a magnetic field. Similar to that for the energy density of the electric field, $u_E = \frac{1}{2}\,\epsilon_0 E^2$.
- SI units of energy density u_B are $\frac{\mathrm{J}}{\mathrm{m}^3}$. This is equivalent to $\frac{\mathrm{N}}{\mathrm{m}^2}$. These units are the same as for <u>pressure</u>.
- Energy densities u_B and u_E naturally appear in the discussion of wave energy and momentum of electromagnetic waves (light).

Mutual Inductance

- Inductance effects not limited to emf effect of a device on itself. Inductance can also occur between different devices. In those cases we refer to there being a mutual inductance.
- Consider two separate, yet nearby coils: magnetic field of one can create a magnetic flux in the other (and vice-versa). Change of the magnetic field in one (by a change in its current) can induce a change in the magnetic flux of the other ⇒ one coil induces emf in the other.





Mutual Inductance (2)

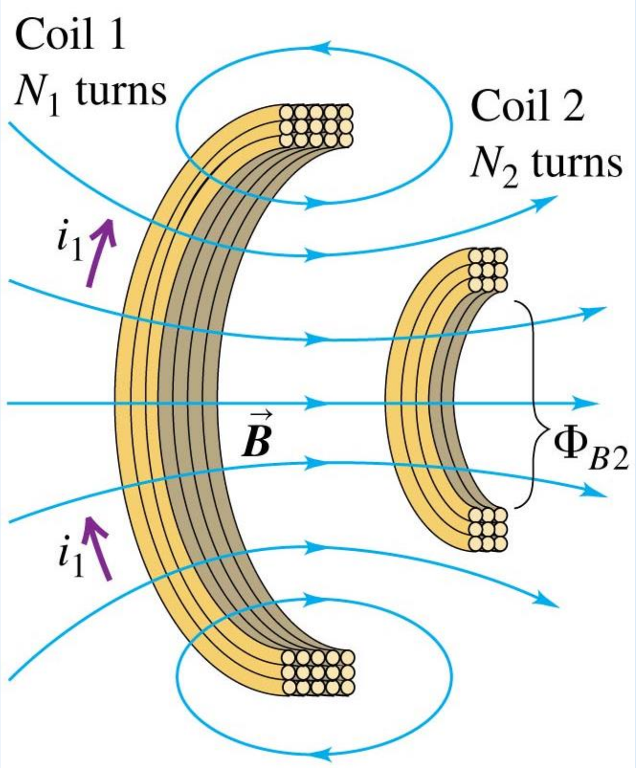
- Consider two neighboring coils of wire. If current in coil 1 changes, it induces an emf in coil 2, and vice versa.
- Proportionality constant for this pair of coils is the <u>mutual inductance</u>, M.
- Define mutual inductance of coil 2 with respect to coil 1 as

$$M_{21} = \frac{N_2 \Phi_{B2}}{i_1},$$

and that of coil 1 with respect to coil 2 as

$$M_{12} = \frac{N_1 \Phi_{B1}}{i_2}.$$

- Being inductances, mutual inductances M_{21} and M_{12} are independent of magnetic field and current in either coil. <u>They depend only on the geometry of the coils and physical constants.</u>



Mutual Inductance (3)

- From their definitions, it follows that coil emfs are

$$\mathcal{E}_1 = -N_1 \frac{d\Phi_{B1}}{dt} = -M_{12} \frac{di_2}{dt} \ ,$$

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{B2}}{dt} = -M_{21} \frac{di_1}{dt} \ .$$

- With a little more detail and math than we will be using in PHYS 1200, one can show from general considerations that there exists a <u>reciprocity relation</u>, such that, for general loop circuits 1 and 2,

$$M_{12} = M_{21} = M \ ,$$

and from which it follows for the magnetically linked coils,

$$\mathcal{E}_1 = -M \frac{di_2}{dt} \ ,$$

$$\mathcal{E}_2 = -M \frac{di_1}{dt} \ .$$