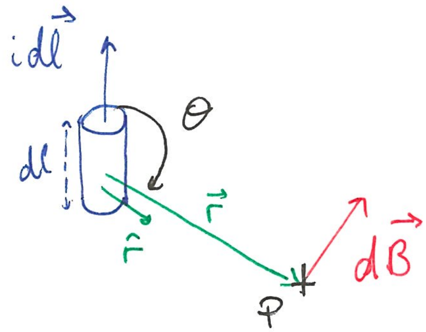
Lecture 13 : Sources of magnetic fields

A careful analysis of electric fields and charge using space and time transformations in special relativity shows that electric and magnetic fields can be unified into one field that transforms between one and the other when observer and charge are in motion relative to one another.
We will not discuss relativistic transformations of fields in this course but you can look forward to it in Electromagnetic Theory.

Historically, the first source of magnetic field equation was deduced by examining the creation of magnetic fields due to currents in conductors, following Oersted's discovery.



In 1820, Biot and Savart found that the generation of a magnetic field increment ($d\vec{B}$) at a point P due to an element of conducting wire (dl) carrying a current i can be expressed as:

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{l} \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi} \frac{i d\vec{l} \times \hat{r}}{r^2}}$$

(also known as the Biot-Savart Law), where $d\vec{l}$ is the direction of the current and all the other symbols have the same meaning as in the point charge form.

- Magnitude of the increment of field is given by

$$\boxed{dB = \frac{\mu_0}{4\pi} \frac{i dl \sin\theta}{r^2}}$$

- Current element and point charge forms of the Biot-Savart law can be seen as equivalent through

$$i dl = \frac{dq}{dt} dl = dq \frac{dl}{dt} = dq\, v$$

for charge increment dq.

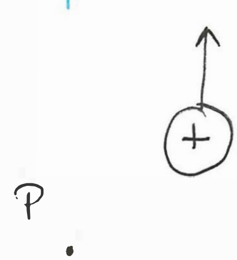
The total magnetic field from a wire is obtained by integrating field contribution from each $d\vec{l}$:

$$\boxed{\vec{B} = \int d\vec{B} = \int \frac{\mu_0}{4\pi} \frac{i\,d\vec{l} \times \hat{r}}{r^2}}$$

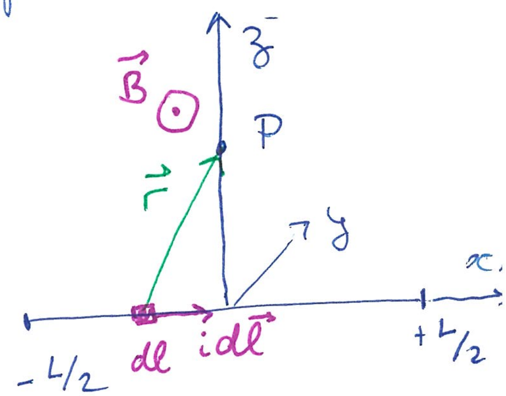
- Direction of the magnetic field: another RHR

→ Point your right thumb along the direction of the current, or the direction of the velocity for a moving positive point charge. The magnetic field loops/lines are then given by the direction that the fingers of your right hand wrap around. If you are considering the motion of a negative charge, point your thumb in the opposite direction of the charge's velocity vector to find the field direction.

Think 13.1: A positive charge moves upward. What is the direction of the magnetic field at point P?



Example: Magnetic field for a straight current carrying wire of length L.
Find the total field at point P a distance z away along the line bisecting the wire.



From the figure: $d\vec{l} \times \vec{r}$ points out of the page at point P

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{l} \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi} \, i dx \, \hat{i} \times (x\hat{i} + z\hat{k})$$

$$= \frac{\mu_0}{4\pi} \frac{i z \, dx}{(x^2+z^2)^{3/2}} (-\hat{j})$$

$$\Rightarrow \vec{B} = \int d\vec{B} = \int_{-L/2}^{+L/2} \frac{\mu_0}{4\pi} iz \frac{dx}{(x^2+z^2)^{3/2}} (-\hat{j})$$

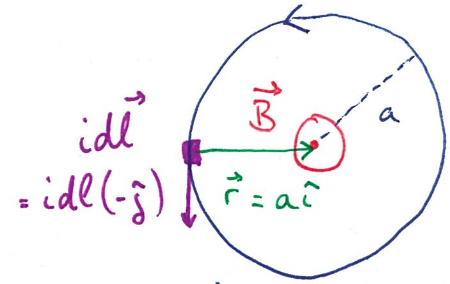
$$\vec{B} = \frac{\mu_0 i}{4\pi} \frac{L}{z\sqrt{(L/2)^2 + z^2}} (-\hat{j})$$

In the limit $L \to \infty$, the magnetic field amplitude becomes

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

where we set $z = r =$ radial distance from wire.

Example: Magnetic field at center of current-carrying wire loop of radius a.



From Biot-Savart and the RHR, the magnetic field at the center is directed out of the page ($\hat{k}$ direction).

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{l} \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi} \frac{(-i dl\, \hat{j} \times a\hat{i})}{a^3}$$

$$= \frac{\mu_0 i dl}{4\pi a^2} \hat{k}$$

Integrating around the entire circular wire, we get

$$\vec{B} = \int_0^{2\pi a} \frac{\mu_0 I}{4\pi a^2} dl\, \hat{k} = \frac{\mu_0 i}{4\pi a^2} (2\pi a)\, \hat{k}$$

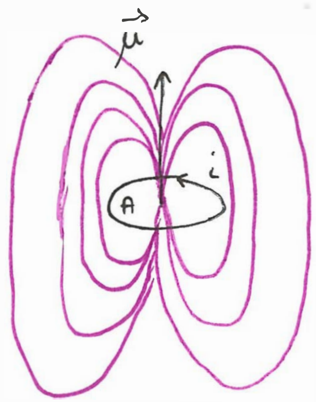
The magnitude of the magnetic field at the center of a circular current-carrying wire is:

$$\boxed{B = \frac{\mu_0 i}{2a}}$$

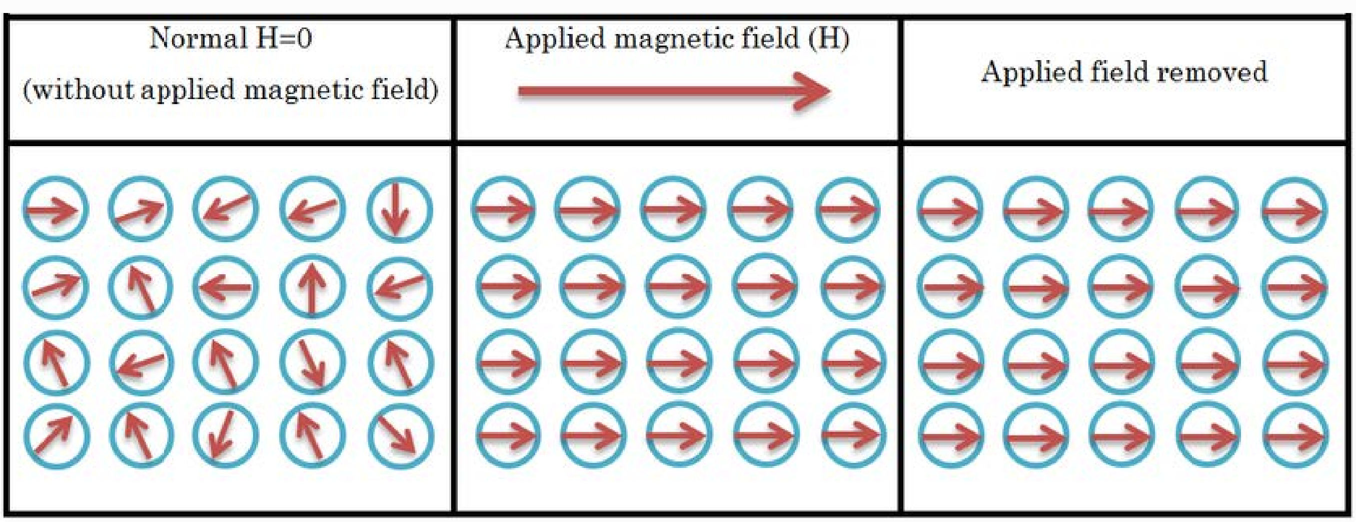
And for an arc of angle ϕ: $B = \frac{\mu_0 i}{2a} \frac{\phi}{2\pi}$

Magnetism in materials

Electrons and other fundamental particles have a magnetic field that can be thought of as a current loop. The field is that of a dipole. In materials, these dipoles interact with one another and with externally applied fields.



In a ferromagnetic material, dipoles align with the applied field and remain aligned when the field is removed.



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